Journal of
Mechanical
Science and
Technology

Journal of Mechanical Science and Technology 21 (2007) 881~884

Nonstationary Random Analysis of Structures with Hybrid Mass Damper System

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(Manuscript Received November 16, 2006; Revised January 8, 2007)

Abstract

The building structure is modeled as a linear single-degree-of-freedom system, and an active mass damper (AMD) is also adopted as the control device in order to reduce the structural response due to the external random load. In addition, suppose that the input acceleration of earthquake can be modeled as the product of nonstationary envelope function and narrow-band stationary random process with Kanai-Tajimi spectrum. In order to obtain the nonstationary stochastic response of control system due to the narrow-band nonstationary random process, in consideration of exchanging the active control mode for the passive one, considerable efforts are generally required in the analytical formulation. This paper, therefore, proposes an analytical technique to acquire the response covariance matrix by utilizing the state transition matrix method in state space. The proposed technique is evaluated by comparing the analytical results with those from numerical simulations.

Keywords: Hybrid mass damper; Nonstationary random analysis

1. Introduction

Recently, there have been increased a number of studies on the active vibration control of structures (for example, ship, automobile, aircraft, etc.) subjectted to external random loads. The major design target of current active vibration control has been limited in many cases to improvement in being more comforttable and the like. In the future, however, it may be considered that it becomes more important to estimate and confirm the structural reliability and safety for the case where the active control technique is utilized for reducing the design load level, or the case where the active control system suddenly works owing to some system troubles at a severe external load even though the normal operation at the severe load is designed so as to exchange the active control mode for the passive one.

When an active control device governed by computer is mounted on a structure, it should be required to estimate the reliability and safety of the control system, though it is not necessary to do so in passive control. However, there are few researches to investigate the structural reliability and/or safety for the actively controlled systems or equipments, because the improvement of the performance and the economy is a priority matter at development. The final target, therefore, in connection with the presenting paper is to implement the probabilistic safety assessment (PSA), which is frequently executed at each nuclear power plant or other extremely important structure, against various types of structure with active control. At the PSA analysis of the actively controlled structure for the use period, there are lots of uncertainties such as occurrence time, duration time and spectral property of each external random

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load, and also occurrence probabilities of failure at software and/or hardware in the control system. Only the analytical approach, therefore, can approximately practice the PSA analysis, since Monte Carlo simulation needs astronomical calculation time owing to the huge combinations of uncertainties. Then, the main purpose of this paper is to present an analytical method in order to obtain the statistical responses of structures with active control system, which may be required at PSA analysis. In this paper, the vibration control of a building structure subjected to earthquake is adopted as a typical example.

2. Mathematical model

Figure 1 shows the mathematical model to estimate the statistical responses of an actively controlled main structure with hybrid method due to earthquake. Although several kinds of active control system have been developed for the aseismic design, this paper adopts one of the simplest systems as shown in Fig. 1, that is Active Mass Damper (AMD) technique, following the hybrid method which means the mechanics in order to exchange the active control mode for the passive one subjected to a large earthquake. Here, it is assumed that only one exchange for the passive mode is occurred during one earthquake.

In Fig. 1, y, y_d and z_0 describe absolute displacements of main structure, AMD and ground, respectively, and m, c and k mean mass, damping coefficient and stiffness of the structure, respectively, and also suffix d gives the parameter relating to AMD, and f shows control force. System equations for the mathematical model shown in Fig. 1 are given as follows:

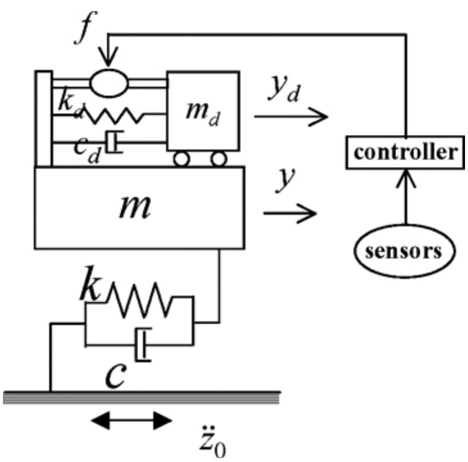

Fig. 1. Mathematical model.

$$m\ddot{y}+c(\dot{y}-\dot{z}_0)+k(y-z_0)+c_d(\dot{y}-\dot{y}_d)+k_d(y-y_d)=-f \tag{1}$$

$$m_d\ddot{y}_d+c_d(\dot{y}_d-\dot{y})+k_d(y_d-y)=f \tag{2}$$

$$f=gu \tag{3}$$

where u is control signal and g means transformed gain from u to f. Suppose that the ground acceleration $\ddot{z}_0$ can be modeled as the products of a deterministic non-stationary function (envelope function) $a(t)$ and a narrow-banded stationary Gaussian random process with Kanai-Tajimi spectrum (Tajimi, 1960) $\ddot{w}(t)$ in the following form.

$$\ddot{z}_0(t)=a(t)\cdot\ddot{w}(t) \quad ; \quad a(t)=A_m\left(e^{-c_1t}-e^{-c_2t}\right) \tag{4}$$

Random process $\ddot{w}(t)$ can be easily obtained by calculating the following equation:

$$\ddot{v}+2h\Omega\dot{v}+\Omega^2v=-\ddot{z}_g \quad ; \quad \left(\ddot{w}=\ddot{v}+\ddot{z}_g\right) \tag{5}$$

where $\ddot{z}_g$ is a stationary Gaussian white noise process of which statistics are described by

$$E\left[\ddot{z}_g(t)\right]=0, E\left[\ddot{z}_g(t)\ddot{z}_g(t+\tau)\right]=2\pi\zeta\delta(\tau) \tag{6}$$

where $E[\ ]$ is the expectation operator, $\delta(\ )$ is the Dirac delta function and ζ is spectral intensity of the random process.

3. Control rule and covariance of structural response

3.1 State space equation

By defining the relative displacements as follows,

$$x=y-z_0,\ x_a=y_d-y \tag{7}$$

and coupling those variables with Eqs. (1), (2) and (3), the following extended state equation related to a hybrid(active/passive) controlled structure can be obtained:

$$\dot{\boldsymbol{X}}=\boldsymbol{A}_0\boldsymbol{X}+\boldsymbol{B}u+\boldsymbol{D}\ddot{w} \tag{8}$$

where

$$X=\begin{bmatrix} x \\ \dot{x} \\ x_a \\ \dot{x}_a \end{bmatrix},\quad A_0=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_0^2 & -2\xi\omega_0 & \mu\omega_d^2 & 2\mu\xi_d\omega_d \\ 0 & 0 & 0 & 1 \\ \omega_0^2 & 2\xi\omega_0 & -\omega_d^2(1+\mu) & -2\xi_d\omega_d(1+\mu) \end{bmatrix}$$

$$B=\begin{bmatrix}0\\-\left(\dfrac{g}{m}\right)\\0\\\dfrac{g}{m}\left(1+\dfrac{1}{\mu}\right)\end{bmatrix},\quad D=\begin{bmatrix}0\\-a(t)\\0\\0\end{bmatrix},\quad \omega_0^2=\left(\frac{k}{m}\right),\ \omega_d^2=\left(\frac{k_d}{m_d}\right),\ \mu=\frac{m_d}{m}$$
$$2\xi\omega_0=\left(\frac{c}{m}\right),\ 2\xi_d\omega_d=\left(\frac{c_d}{m_d}\right)$$

4. Control rule

To achieve the active vibration control, LQG algorithm is employed in this paper, and the derivation of control gain is approximately carried out under the steady state conditions. Then a feedback gain vector $\boldsymbol{F}$ is obtained as:

$$u=-\boldsymbol{FX},\ \boldsymbol{F}=r^{-1}\boldsymbol{B}^T\boldsymbol{P} \tag{9}$$

where $\boldsymbol{P}$ is solution of the Riccati equation

$$\boldsymbol{Q}-r^{-1}\boldsymbol{PBB}^T\boldsymbol{P}+\boldsymbol{A}_0^T\boldsymbol{P}+\boldsymbol{PA}_0=\boldsymbol{0} \tag{10}$$

and $\boldsymbol{Q}, r$ are the weighting parameters in the following performance index:

$$J=E\left[\boldsymbol{X}^T\boldsymbol{QX}+ru^2\right] \tag{11}$$

By substituting Eq. (9) for Eq. (8), the revised state equation is given as

$$\dot{\boldsymbol{X}}=\left(\boldsymbol{A}_0-\boldsymbol{BF}\right)\boldsymbol{X}+\boldsymbol{D}\ddot{w}\equiv\boldsymbol{AX}+\boldsymbol{D}\ddot{w} \tag{12}$$

4.1 Derivation of covariance matrix of structural response

Noting that initial values of state variables at t=0 are zero, the general solution of Eq. (12) is well-known as

$$\boldsymbol{X}(t)=\int_0^t\boldsymbol{\Phi}(t-\tau)\boldsymbol{D}(\tau)\ddot{w}(\tau)d\tau \tag{13}$$

where $\boldsymbol{\Phi}(t)$ means the state transition matrix of this system. Taking the expectation as for Eq. (13), then

$$E\left[\boldsymbol{X}(t)\right]=0 \tag{14}$$

is obtained owing to $E\left[\ddot{w}(t)\right]=0$ coming from Eq. (6). In consideration of Eq. (14), the covariance matrix of structural response $\boldsymbol{V}(t)$ is expressed as follows:

$$\boldsymbol{V}(t)=E\left[\boldsymbol{X}(t)\boldsymbol{X}^T(t)\right] \tag{15}$$

and also operating the derivative against Eq. (15) can generate the differential equation such as:

$$\begin{aligned}\dot{\boldsymbol{V}}(t)=\boldsymbol{AV}(t)+\boldsymbol{V}(t)\boldsymbol{A}^T+\left\{E\left[\ddot{w}(t)\boldsymbol{X}(t)\right]\boldsymbol{D}^T(t)\right\}^T\\+E\left[\ddot{w}(t)\boldsymbol{X}(t)\right]\boldsymbol{D}^T(t)\end{aligned} \tag{16}$$

Consider the fourth term at right side in Eq. (16), denoted by $\boldsymbol{\Psi}$, in detail. That is generally explained as follows:

$$\boldsymbol{\Psi}=\int_0^t\boldsymbol{\Phi}(t-\tau)E\left[\ddot{w}(t)\ddot{w}(\tau)\right]\boldsymbol{D}(\tau)\boldsymbol{D}^T(t)d\tau \tag{17}$$

Here, $\ddot{w}(t)$ is modeled as the narrow-banded stationary Gaussian process, and also generated from Eq. (5), therefore, the analytical form of $E\left[\ddot{w}(t)\ddot{w}(\tau)\right]$ has been already evaluated by many researches (for example, Casciati *et al.*, 1991) in the following form.

$$\begin{aligned}E\left[\ddot{w}(t)\ddot{w}(\tau)\right]&=R_{\ddot{w}}(t,\tau)=R_{\ddot{w}}(t-\tau)\\&=R_{\ddot{w}}(\eta);\ (\ \bar{\Omega}=\Omega\sqrt{1-h^2}\ )\\&=\frac{\pi\zeta\Omega}{2h}e^{-h\Omega\eta}\left\{\left(1+4h^2\right)\cos\bar{\Omega}\eta+\frac{h\Omega}{\bar{\Omega}}\left(1-4h^2\right)\sin\bar{\Omega}\eta\right\}\end{aligned} \tag{18}$$

Thus, Eq. (17) becomes

$$\boldsymbol{\Psi}=\int_0^t\boldsymbol{\Phi}(\eta)R_{\ddot{w}}(\eta)\boldsymbol{D}(t-\eta)\boldsymbol{D}^T(t)d\eta \tag{19}$$

and therefore, non-stationary covariance matrix of structural response can be eventually estimated by solving Eq. (16) in consideration of Eq. (19).

Since Eq. (16) is resolved numerically, the phenomenon exchanging the active control mode for the passive one can be easily calculated by changing some parameter values at such an event occurrence time.

5. Estimation of response peak factor

The relative displacement of AMD, x_a, is placed as a standard parameter in order to judge the exchange of active/passive mode, in this paper. Hence, the peak factor of x_a should be evaluated , because all the calculated results are response covariance as statistical values. Peak factor proposed by Kiureghian (1980) p_{x_a} is, in this research, adopted under the assumption that the peak factor for gentle non-stationary process is nearly equal to one of stationary process. Then, the occurrence time of exchange is approximately decided so that the standard deviation of x_a, $\sigma_{x_a}(t)$, arrives at $(\tilde{x}_a/p_{x_a})$, where $\tilde{x}_a$ means a maximum value of x_a during active control mode.

In order to acquire the peak factor proposed by Kiureghian, the following *m-th* spectral moment related to x_a should be required finally:

$$\lambda_m = \int_0^\infty \omega^m G_b(\omega) \left| H_a(\omega) \right|^2 d\omega \tag{20}$$

where $G_b(\omega)$ shows one-sided power spectral density of $\ddot{w}(t)$, and $H_a(\omega)$ means complex frequency response function being transferred from $\ddot{w}(t)$ to x_a. By utilizing Equation (5), $G_b(\omega)$ can be given as follows:

$$G_b(\omega) = 2\zeta \frac{\Omega^4 + (2h\Omega\omega)^2}{\left\{(\Omega^2 - \omega^2)^2 + (2h\Omega\omega)^2\right\}} \tag{21}$$

and also $H_a(\omega)$ can be analytically obtained by using Eq. (12).

6. Numerical Examples

Some numerical results are shown to illustrate the validity of the proposed technique. The following set of parameters are used for numerical calculations.:

$$\omega_0 = 6\pi,\ \xi = 0.01,\ \omega_d = 0.99\omega_0,\ \xi_d = 0.09\xi,$$
$$\Omega = 5\pi,\ h = 0.6$$
$$(g/m) = 0.98,\ \mu = 0.02,\ \zeta = 50,\ c1 = 0.3,$$
$$c2 = 0.5,\ A_m = 5.38$$
$$\tilde{x}_a = 7\ ,\ Q = diag[100, 100, 1, 1]\ ,\ r = 1$$

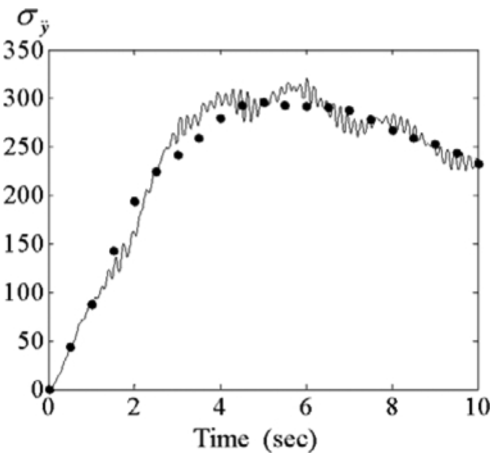

Fig. 2. Results for hybrid control.

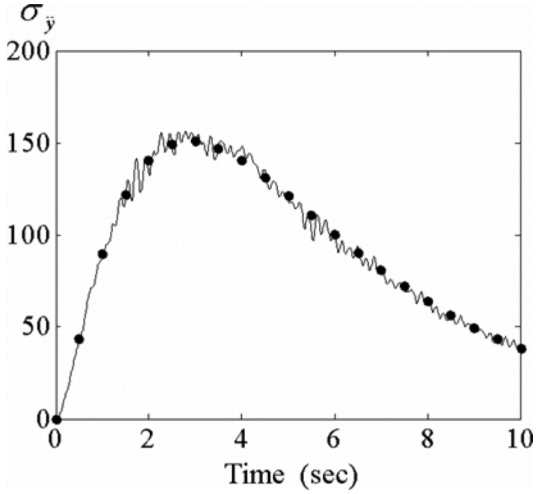

Fig. 3. Results for active control.

Figures 2 and 3 show the non-stationary standard deviations of absolute acceleration of main structure, $\sigma_{\ddot{y}}$, for the cases of hybrid control method and full active one, respectively. Black circles in each figure give the analytical results, and solid line means those by Monte Carlo simulations with four hundred times.

Both results show good agreements in each figure.

7. Conclusions

Analytical results are compared with numerical simulations, and both show a good agreement. As a result, it seems that the validity of the proposed technique is confirmed. It is considered that the newly developed technique is useful for the seismic PSA analysis for a hybrid or active controlled structure.

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